

Fundamentals of Mathematics

1) PP-2

Q. Prod of all sol. of $x^{\log_{10} x} = (100 + 2^{\sqrt{\log_2 3} - 3^{\sqrt{\log_3 2}}}) x$ is

$$2^{\sqrt{3}} = (\sqrt{2} \cdot \sqrt{2})^{\sqrt{3}} = \sqrt{2}^{\sqrt{3}} \cdot \sqrt{2}^{\sqrt{3}}$$

$$\begin{aligned} x^{\log_{10} x} &= 100x \\ \log_{10} x^{\log_{10} x} &= \log_{10} (100x) \\ \log_{10} x \cdot \log_{10} x &= \log_{10} 100 + \log_{10} x \\ t \cdot t &= 2 + t \\ t^2 - t - 2 &= 0 \Rightarrow (t-2)(t+1) = 0 \end{aligned}$$

Note

$$a^{\sqrt{\log_a b}} = b^{\sqrt{\log_b a}}$$

Try Prove

$$\begin{aligned} t &= 2 & \& t &= -1 \\ \log_{10} x &= 2 & \log_{10} x &= -1 \\ x &= 10^2 & x &= 10^{-1} \\ &= 100 & &= \frac{1}{10} \end{aligned}$$

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Q2 If $\log_7 2 = m$ then $\log_{49} 28$ is

$$2(1+2m) \quad \text{or} \quad \frac{1+2m}{2} \quad \frac{2}{1+2m} \quad 1+m.$$

$$\log_{49} 28 = \log_{7^2} 7 \times 2^2 = \frac{1}{2} \log_7 (7 \times 2^2)$$

$$3^{\cancel{5}} = 405$$

$$3^c = 3^4 \Rightarrow \boxed{c=4}$$

$$= \frac{1}{2} (\log_7 7 + \log_7 2^2)$$

$$= \frac{1}{2} (1 + 2 \log_7 2) = \frac{1}{2} (1 + 2m)$$

Q4 If $p = \log_5 (\log_5 3)$ & $3 = 405$ then $(\quad)^{+5^{-p}}$

$$3^{(+5^{-p})} = 405$$

$$a^{m+n} = a^m \times a^n$$

$$\Rightarrow 3^c \times 3^{5^{-p}} = 405 \Rightarrow 3^c \cdot 3^{\log_5 5} = 405$$

$$5^{-p} = 5^{-(\log_5 (\log_5 3))} = 5^{\log_5 (\log_5 3)^{-1}}$$

$$= (\log_5 3)^{-1} = \frac{1}{\log_5 3}$$

$$= \log_3 5$$

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Q. If $\frac{a + \log_4 3}{a + \log_2 3} = \frac{a + \log_8 3}{a + \log_4 3} = b$ then $b = ?$

$$\frac{a}{b} = \frac{c}{d} = \frac{c-a}{d-b}$$

$$\frac{m}{n} = \frac{x}{y} = \frac{x-m}{y-n}$$

$$= \frac{(a + \log_8 3) - (a + \log_4 3)}{(a + \log_4 3) - (a + \log_2 3)} = b$$

$$b = \frac{\log_2 3 - \log_2 3}{\log_2 3 - \log_2 3} = \frac{\frac{1}{3} \log_2 3 - \frac{1}{2} \log_2 3}{\frac{1}{2} \log_2 3 - \log_2 3} = \frac{\log_2 3 \left(\frac{1}{3} - \frac{1}{2} \right)}{\log_2 3 \left(\frac{1}{2} - 1 \right)}$$

$$b = \frac{\frac{1}{3} - \frac{1}{2}}{\frac{1}{2} - 1} = \frac{+\frac{1}{6}}{+\frac{1}{2}} = \frac{1}{3}$$

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Σ Ka mtlb r Ki Value ek ek Kr K chdao 8 + + Lgaate Juo
 ↑ 2400 1023 2011
 Δ x x x

$$L = \sum_{r=7}^{2400} \log_7 \left(\frac{r+1}{r} \right), \quad M = \bigwedge_{r=2}^{1023} \log_8 (r+1), \quad N = \sum_{r=2}^{2011} \frac{1}{\log_r P} \quad \left\{ P = 1 \cdot 2 \cdot 3 \cdot 4 \dots 2011 \right\}$$

$$L + M = 13 = 3 + 10$$

$$M^2 + N^2 = 101 \quad 10^2 + 1^2 = 101$$

$$3 - 10 + 1 \neq 6$$

$$L M N = 30 \quad 3 \times 10 \times 1$$

$$L = \log_7 \frac{7+1}{7} + \log_7 \frac{8+1}{8} + \log_7 \frac{9+1}{9} + \dots + \log_7 \frac{2400+1}{2400}$$

$$= \log_7 \frac{8}{7} + \log_7 \frac{9}{8} + \log_7 \frac{10}{9} + \dots + \log_7 \frac{2401}{2400}$$

$$N = \frac{1}{\log_2 P} + \frac{1}{\log_3 P} + \frac{1}{\log_4 P} + \dots + \frac{1}{\log_{2011} P}$$

$$N = \log_P 2 + \log_P 3 + \log_P 4 + \dots + \log_P 2011$$

$$N = \log_P (2 \times 3 \times 4 \times \dots \times 2011) = \log_P P = 1$$

$$L = \log_7 \left\{ \frac{8}{7} \times \frac{9}{8} \times \frac{10}{9} \times \frac{11}{10} \times \dots \times \frac{2401}{2400} \right\}$$

$$L = \log_7 \frac{2401}{7} = \log_7 7^3 = 3$$

$$M = \log_2 (2+1) \times \log_3 (3+1) \times \log_4 (4+1) \times \log_5 (5+1) \dots \log_{1023} (1023+1)$$

$$M = \log_2 3 \times \log_3 4 \times \log_4 5 \times \log_5 6 \dots \log_{1023} 1024$$

$$M = \frac{\log 1024}{\log 2} = \log_2 1024 = \log_2 2^{10} = 10$$

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Q11 $\prod_{r=3}^{26} \log_r(r+1) = 3^x$ find x ?

$$\log_3(3+1) \times \log_4(4+1) \times \dots \times \log_{26}(26+1)$$

$$\log_3 4 \times \log_4 5 \times \log_5 6 \dots \log_{26} 27 = 3^x$$

$$\frac{\log 4}{\log 3} \times \frac{\log 5}{\log 4} \times \frac{\log 6}{\log 5} \dots \frac{\log 27}{\log 26}$$

$$\log 27 = 3^x \Rightarrow \log_{10} 3^3 = 3^x$$

$$\Rightarrow 3 = 3^x$$

$$\Rightarrow x = 1$$

Q K is unique +ve value satisfying

Eqⁿ $(4K)^{\log 2} = (9K)^{\log 3}$ then $72K$?

$$\log_{10}(4K)^{\log 2} = \log_{10}(9K)^{\log 3}$$

$$\log 2 \log(4K) = \log 3 \log(9K)$$

$$\log 2 (\log 4 + \log K) = \log 3 (\log 9 + \log K)$$

$$\log 2 \log 2^2 + \log 2 \log K = \log 3 \log 3^2 + \log 3 \log K$$

$$2(\log 2)^2 + \log 2 \log K = 2(\log 3)^2 + \log 3 \log K$$

$$2(\log 2)^2 - 2(\log 3)^2 = \log K (\log 3 - \log 2)$$

$$2(\log 2 - \log 3)(\log 2 + \log 3) = \log K (\log 3 - \log 2)$$

$$-2(\log 2 + \log 3) = \log K$$

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$$(\log 2)^2 \neq \log 2^2$$

$$Q \quad x^{(1+\log x)} = 10x \text{ find } x?$$

$$\log_{10} x^{(1+\log_{10} x)} = \log_{10} (10x)$$

$$(1+\log_{10} x) \log_{10} x = \log_{10} 10 + \log_{10} x$$

$$(1+t)t = (1+t)$$

$$(1+t)(t-1) = 0$$

$$t = -1, t = 1 \Rightarrow \log_{10} x = -1 \mid \log_{10} x = 1$$

$$x = 10^{-1}$$

$$x = 10$$

$$Q \quad x^{\frac{\log x + 5}{2}} = 10^{5+\log x} \text{ find } x?$$

Fundamentals of Mathematics

Questions Based on Tricotomy.

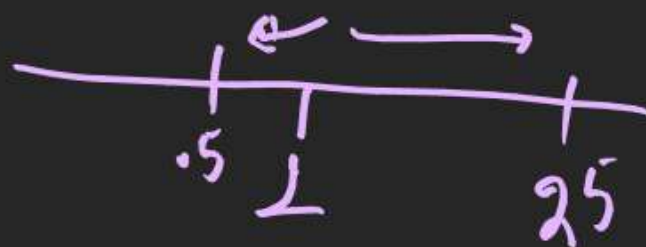
Which is greater?

(1) $A = \log_{.5} 25$, $B = \log_2 3$
 $.5 < 0$, > 0

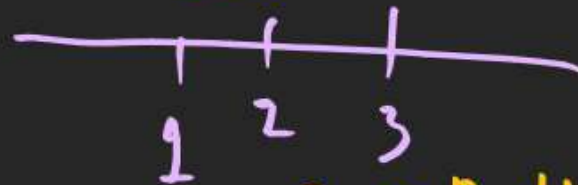
Check Base & exponent WRT 1

If Both are on Same Side of 1
 then log is +ve otherwise -ve

$A = \log_{.5} 25 = -ve$



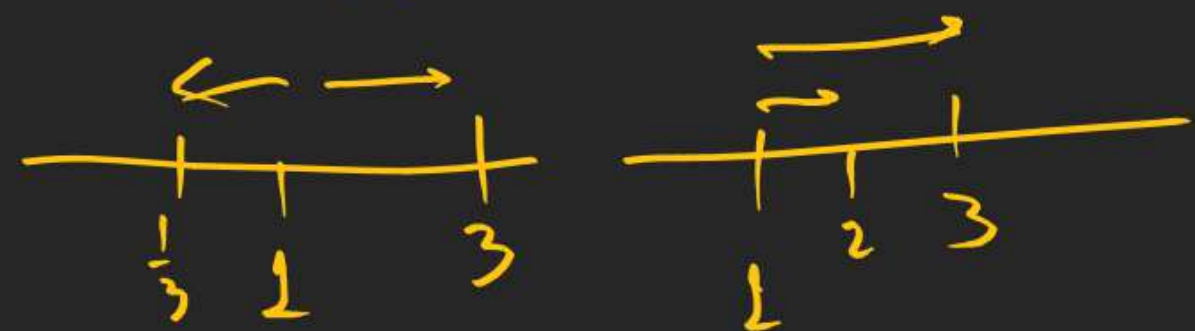
$B = \log_2 3 = +ve$



3, 2 Both are on Same Side

As $A \text{ is } < 0$ & $B \text{ is } > 0$
 So $B > A$

Q $A = \log_{\frac{1}{3}} 3$ & $B = \log_2 3$
 $= -ve$, $= +ve$



$B > A$

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$$\log_3 3 = 1 \quad \log_3 4 > 1$$

On Behalf of values hr. thum.

Q $A = \log_{\boxed{3}}^{\boxed{3.1}}$ & $B = \log_{3.1}^{\boxed{3}}$

Which is hr.

$$\text{Exp} > \text{Base}$$

then $\log \text{value} > 1$

$$A > B$$

$$\text{Exp} = 3$$

$$\text{Base} = 3.1$$

$$\Rightarrow \text{Exp} < \text{Base}$$

$$B = \log \text{value} < 1$$

$$A = \log_3 4$$

$$\text{Exp} = 4$$

$$\text{Base} = 3$$

$$\text{Exp} > \text{Base}$$

$$\therefore A = \log_3 4 > 1$$

$$B = \log_4 3$$

$$\text{Exp} = 3$$

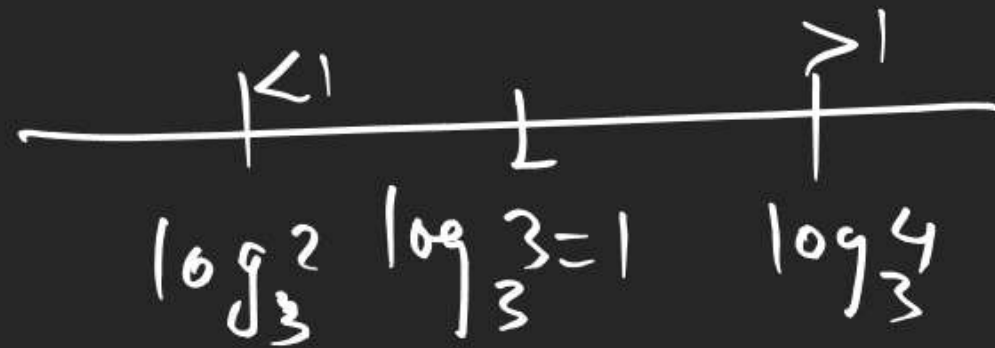
$$\text{Base} = 4$$

$$\text{Exp} < \text{Base}$$

$$B = \log_4 3 < 1$$

$$A > B$$

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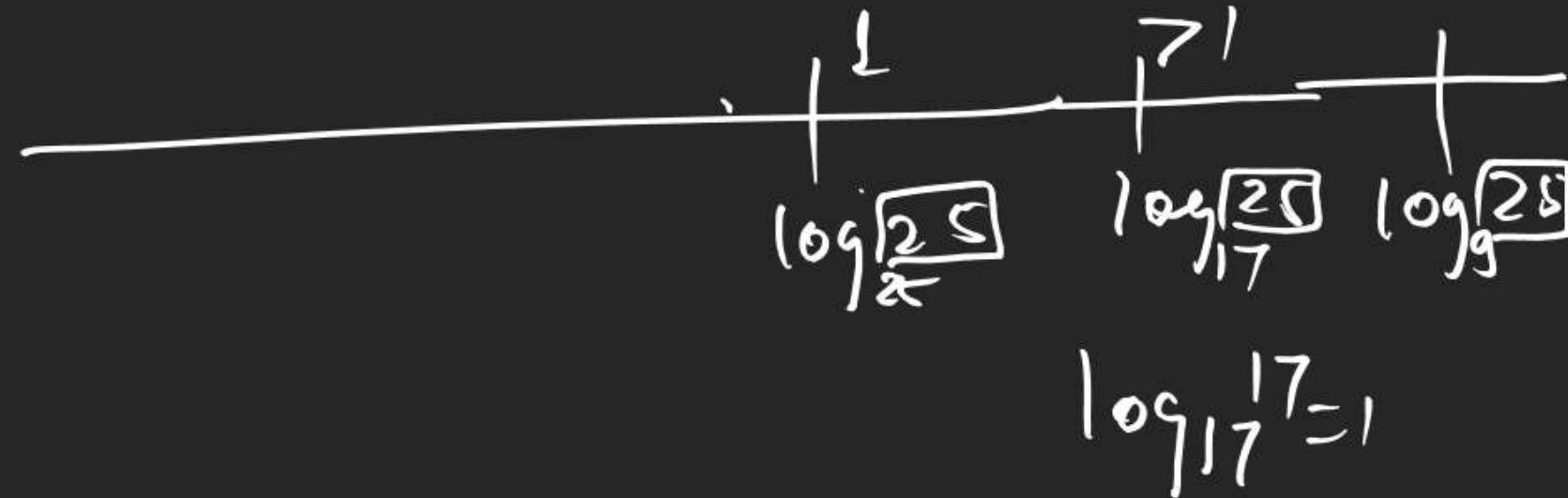


Q $A = \log_3 5$ & $B = \log_{17} 25$

Which is greater?

$A = \frac{\log 25}{\log 3}$ | $B = \log_{17} 25$

$A > B$



$\log_{17} 17 = 1$



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Some Imp. pt.

$$(1) \log_e e = 1$$

$$(2) \log_e x = \ln x$$

$$(3) \log_{10} 3 = .4771$$

$$(4) \log_{10} 2 = .3010$$

$$(5) \ln 2 = .693$$

$$\log_{10} 34.56 = 1.5386$$

$$= 1 + .5386$$

Int.
Part

fractional
Part

Mantissa.

(characteristic)

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2 New fns.

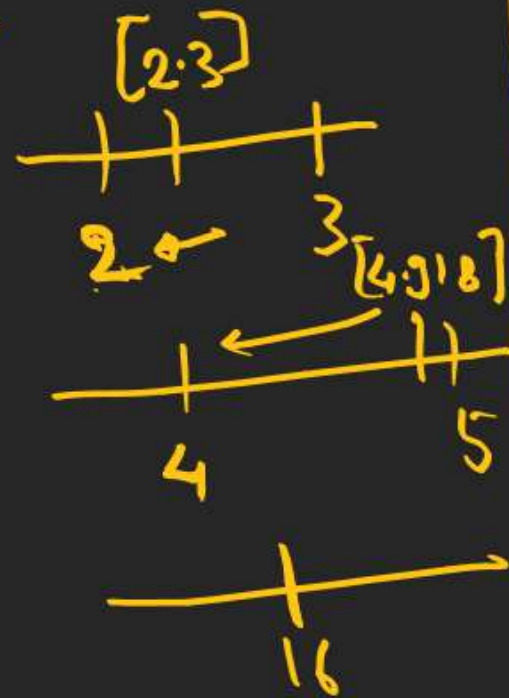
① Greatest Integer fn.

1) $f(x) = [x]$ Rep.

2) $[2.3] = 2$

$[4.918] = 4$

$[16] = 16$



$[-17] = -17$

$[-17.4] = -18$

$[-2023.29] = -2024$

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(3)

$$[x] = 4$$

$$x \in [4, 5)$$

$$[x] = 7$$

$$x \in [7, 8)$$

$$[x] = 29$$

$$x \in [29, 30)$$

final $[x] = n$

$$x \in [n, n+1)$$

$$[x] = -11$$

$$x \in [-11, -11+1)$$

$$x \in [-11, -10)$$

Q

$$[\log_{10} x] = 4$$

then $x \in ?$

$$\log_{10} x \in [4, 5)$$

$$4 \leq \log_{10} x < 5$$

$$10^4 \leq x < 10^5$$

$$10000 \leq x < 100000$$

$$x \in [10000, 100000)$$